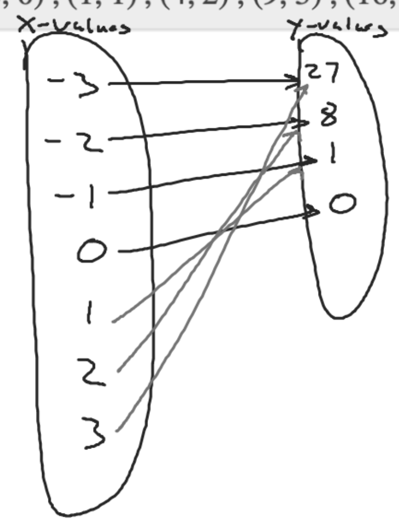
ONE-TO-ONE FUNCTION

A function is **one-to-one** if each value in the range corresponds to one element in the domain. For each ordered pair in the function, each y-value is matched with only one x-value. There are no repeated y-values.

For each set of ordered pairs, determine if it represents a function and, if so, if the function is one-to-one.

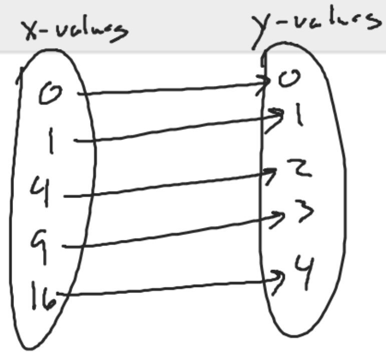
ⓐ $\{(-3,27),(-2,8),(-1,1),(0,0),(1,1),(2,8),(3,27)\}$ and ⓑ $\{(0,0),(1,1),(4,2),(9,3),(16,4)\}$.

a)



Yes Function

Not one-to-one



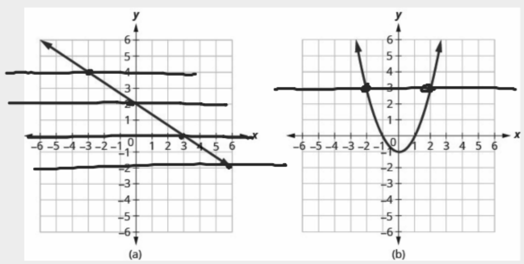
Yes Function

Yes one-to-one

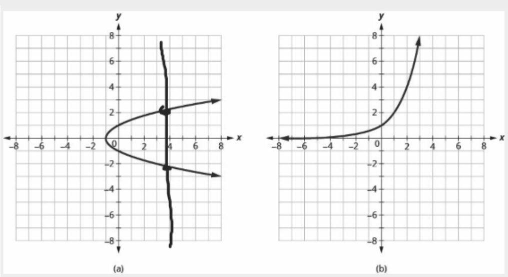
For each set of ordered pairs, determine if it represents a function and if so, is the function one-to-one.

ⓐ $\{(27, -3), (8, -2), (1, -1), (0, 0), (1, 1), (8, 2), (27, 3)\}$ ⓑ $\{(7, -3), (-5, -4), (8, 0), (0, 0), (-6, 4), (-2, 2), (-1, 3)\}$

Determine ⓐ whether each graph is the graph of a function and, if so, ⓑ whether it is one-to-one.



(a) (b)



(a) (b)

INVERSE OF A FUNCTION DEFINED BY ORDERED PAIRS

If $f\,(x)$ is a one-to-one function whose ordered pairs are of the form $(x,\,y)$, then its inverse function $f^{-1}\,(x)$ is the set of ordered pairs $(y,\,x)$.

Switch x + y

Inverse

Find the inverse of the function $\{(0,3)\,,(1,5)\,,(2,7)\,,(3,9)\}$. Determine the domain and range of the inverse function.

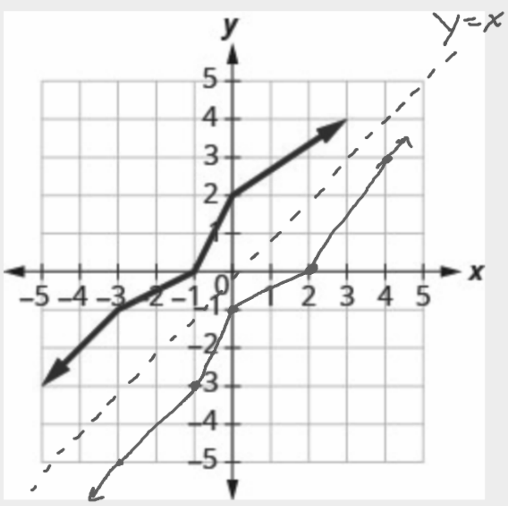
$f^{-1}\,\{(3,0)\ (5,1)\ (7,2)\ (9,3)\}$

D: $\{3, 5, 7, 9\}$

R: $\{0, 1, 2, 3\}$

$f(x)$ D R $f^{-1}(x)$ D: R:

Graph, on the same coordinate system, the inverse of the one-to one function shown.



$f(x)$ (0,2) (-1,0) (-3,-1) (3,4)
(-5,-3)

$f^{-1}(x)$ (2,0) (0,-1) (-1,-3) (4,3)
(-3,-5)

INVERSE FUNCTIONS

$$\begin{aligned} f^{-1}(f(x)) &= x, \text{ for all } x \text{ in the domain of } f \\ f\left(f^{-1}(x)\right) &= x, \text{ for all } x \text{ in the domain of } f^{-1} \end{aligned}$$

$(f \circ g)(x) = (g \circ f)(x)$

x

Verify that $f(x) = 5x - 1$ and $g(x) = \frac{x+1}{5}$ are inverse functions.

$(f \circ g)(x)$

$f(g(x)) = 5x-1$

$f\left(\frac{x+1}{5}\right) = 5x-1$

$\cancel{5}\left(\frac{x+1}{\cancel{5}}\right)-1$

$x+1-1$

x

$(g \circ f)(x)$

$g(f(x)) = \frac{x+1}{5}$

$g(5x-1) = \frac{x+1}{5}$

$= \frac{5x-1+1}{5}$

$= \frac{5x}{5}$

$= x$

Verify that the functions are inverse functions.

$f(x) = 2x + 6$ and $g(x) = \frac{x-6}{2}$.

$f(g(x)) = 2x+6$

$f\left(\frac{x-6}{2}\right) = \cancel{2}\left(\frac{x-6}{\cancel{2}}\right)+6$

$= x-6+6$

$= x$

$g(f(x)) = \frac{x-6}{2}$

$g(2x+6) = \frac{2x+6-6}{2}$

$= \frac{2x}{2}$

$= x$

1, 3, 5, 9, 11
13, 17-20
21, 23, 27, 29
31-37 odd